$$1+s_n^4=u_n^2\ldots\ldots(4),$$

whence $u_n^2-s_n^4=1$, and $u_n=1$ and $s_n=0$ are the only solutions. Now s_n cannot be 0 for if such were the case we would have

$$s_n=s_{n-1}=s_{n-2}\ldots\ldots=s_1=x=0,$$

which is inconsistent with the definition of x. Hence the impossibility of (4) and therefore of (1) is completely demonstrated.

134. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

Solve neatly and briefly the equations

$$x^3+x^2y+y^3=53\ldots.(1),\ y^3+y^3z+z^3=13\ldots.(2),\ \text{and}\ z^3+z^2x+x^3=31\ldots.(3).$$

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Probably the only brief solution is by inspection, as follows:

$$x^3+x^2y+y^3=53=27+18+8=3^3+2.3^2+2^3.$$

$$y^3+y^2z+z^3=13=8+4+1=2^3+1.2^2+1^3.$$

$$z^3+z^2x+x^3=31=1+3+27=1^3+3.1^2+3^3.$$

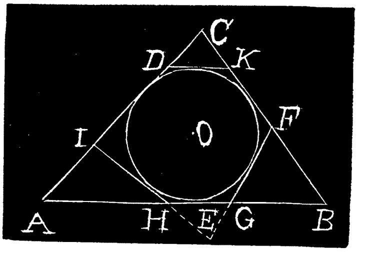
$\therefore$ $x=3$, $y=2$, $z=1$.

135. Proposed by CHARLES C. CROSS, Whaleyville, Va.

Tangents parallel to the three sides are drawn to the in-circle. If p, q, r, be the lengths of the parts of the tangents within the triangle, prove that

$$\frac{p}{a}+\frac{q}{b}+\frac{r}{c}=1.$$

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.



Let ABC be any triangle with the in-circle O. Put $AB=c$, $AC=b$, $BC=a$.

Draw the respective tangents, $DK=r$, parallel to AB; $FG=q$, parallel to AC; and $HI=p$, parallel to BC.

Let $AH=x$, and $BG=y$.

The construction of lines and similarity of triangles give the following:

$HG=DE=r$; $y:c=q:b$, or $y=cq/b$; and $x:c=p:a$, or $x=cp/a$. But $x+y+r=c$.

$$\therefore \frac{cp}{a}+\frac{cq}{b}+r=c;\ \text{or}\ \frac{p}{a}+\frac{q}{b}+\frac{r}{c}=1.$$

Solved in a similar manner by *LON C. WALKER* and *J. SCHEFFER*.

II. Solution by MARCUS BAKER, U. S. Geological Survey, Washington, D. C.

Designate the perpendiculars from A, B, C by h', h'', h'''; then from similar triangles,

$$p':a::h'-2r:h', \text{ or } \frac{p'}{a}=1-2\frac{r}{h'}.$$

Similarly, $\frac{p''}{b}=1-2\frac{r}{h''}$ and $\frac{p'''}{c}=1-2\frac{r}{h'''}$; whence

$$\frac{p'}{a}+\frac{p''}{b}+\frac{p'''}{c}=3-2\left(\frac{r}{h'}+\frac{r}{h''}+\frac{r}{h'''}\right)=1\ldots\ldots(1),$$

$$\left(\text{since, } \frac{1}{h'}+\frac{1}{h''}+\frac{1}{h'''}=\frac{1}{r}\right).$$

Similarly, if P', P'', P''', denote the length of the tangent to the *escribed* circles

$$\frac{P'}{a}+\frac{P''}{b}+\frac{P'''}{c}=3+2\left(\frac{r'}{h'}+\frac{r''}{h''}+\frac{r'''}{h'''}\right)\ldots\ldots(2).$$

Again, for a *tetrahedron*, let p', p'', p''', p'''', designate the planes drawn tangent to the inscribed sphere and parallel to the faces a, b, c, d; and P', P'', P''', P'''' the plane similarly drawn tangent to the escribed sphere. Then

$$\sqrt{\frac{p'}{a}}+\sqrt{\frac{p''}{b}}+\sqrt{\frac{p'''}{c}}+\sqrt{\frac{p''''}{d}}=4-2\left(\frac{r}{h'}+\frac{r}{h''}+\frac{r}{h'''}+\frac{r}{h''''}\right)=2\ldots\ldots(3).$$

$$\sqrt{\frac{P'}{a}}+\sqrt{\frac{P''}{b}}+\sqrt{\frac{P'''}{c}}+\sqrt{\frac{P''''}{d}}=4+2\left(\frac{r'}{h'}+\frac{r''}{h''}+\frac{r'''}{h'''}+\frac{r''''}{h''''}\right)\ldots\ldots(4).$$

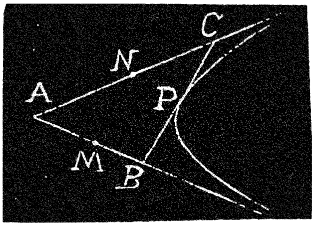
Also solved by *G. B. M. ZERR* and *H. C. WHITAKER*.

GEOMETRY.

164. Proposed by J. M. HARCOURT, M. D., 305 Clinton Street, Brooklyn, N. Y.

Given two tangents to a parabola, find the locus of the center of the nine-point circle of the triangle by the two given tangents and any third tangent.

Solution by J. SCHEFFER, A. M., Hagerstown, Md.



Let the equation of the parabola be $y^2=4mx$; then will the equation of three tangents be of the form $y=ax+\frac{m}{a}\ldots\ldots(1)$, $y=a'x+\frac{m}{a'}\ldots\ldots(2)$,

$y=a''x+\frac{m}{a''}\ldots\ldots(3)$; the equation (1) being that of AB, (2) of AC, and (3) of BC.

The equation of the altitude from A upon